Preface to second edition

A Teaching Challenge:

How to Teach Quantum Mechanics Fully with Engineering Mechanics

• Motivation

With advances in material synthesis and device processing capabilities, the importance of quantum mechanics in applied disciplines, such as material science, electrical engineering and applied physics, has dramatically increased over the last couple of decades. The engineers can no longer just work with simplistic phenomenological equations, but must understand a more fundamental origin of the phenomena. Devices such as Josephson junctions, semiconductor lasers, transistors, and all of the nanostructures cannot be fully understood in terms of simple classical mechanics. However, the current engineering education, especially undergraduate, in NCKU like most of the universities in the world still focuses on classical mechanics that applies conventional Newtonian approaches to solve static and dynamic problems appearing in the macroscopic world. Since its foundation, NCKU is very proud of its engineering education. To continue this excellent tradition toward the coming era of nanotechnology and biotechnology, and to become one of the world leading universities in engineering research and education, our engineering education has to incorporate the fresh elements from quantum mechanics into the current teaching in engineering mechanics.

• Challenge

The big challenge does not stem from our ignorance of the importance of nourishing engineering education with quantum elements, but from the seemingly insurmountable task of teaching quantum mechanical concept in a class of engineering mechanics. The fundamental ideas of quantum mechanics and engineering mechanics are so conflicting that all the deterministic rules and causal trajectories found in engineering mechanics are totally invalid in quantum mechanics, which treats physical quantities as random variables having only probabilistic nature. Some lecturers of quantum mechanics even suggest students to temporarily forget the impressions of determinism and causality gained from classical mechanics, when studying quantum mechanics. Furthermore, because quantum mechanics does not possess equations of dynamics, when one learns quantum mechanics, one has to develop one's own personal sense or picture to interpret the theory. On the other hand, in engineering mechanics the equations of motion can be derived from either Newton formulation or Hamilton formulation or Lagrange formulation. It is safe to say that the lack of equations of motion has been the primary source of controversies about the interpretations of quantum mechanics, although it has the equation of propagation of probability, i.e., the Schrödinger equation. Accordingly, it seems that the knowledge and experience gained from studying engineering mechanics cannot be conveyed directly to the study of quantum mechanics. These remarkable gaps between quantum mechanics and engineering mechanics have forbidden the possibility to teach quantum mechanics in a course in engineering mechanics.

• Aim

The aim of this lecture is to surmount the mentioned obstacles and to propose a teaching program called engineering quantum mechanics which unifies the education of engineering mechanics and quantum mechanics. The theoretical background of the proposed teaching program is based on my recent progress in developing a new theory of quantum mechanics called complex quantum Hamilton mechanics (or complex mechanics, by short). Within the framework of this new formulation, the equations of motion compatible with the Schrödinger equation are proved to be the classical Hamilton equations extended to a complex domain. By employing the complex-extended Hamilton equations or the equivalent Newton equations, there will be no ambiguity at all in understanding the motion of quantum particles; in other words, we no longer need the so-called "interpretation". Everything becomes transparent and every quantum mechanical problems can be solved by the well-established methods in the classical mechanics. From the point of view of complex mechanics, undergraduate students finishing the courses of engineering mechanics and engineering mathematics have already gathered all the required ability and tools to learn quantum mechanics. In the following, the fundamentals of complex mechanics will be introduced first, and then used to construct the main structure of engineering quantum mechanics.

• Literature Survey

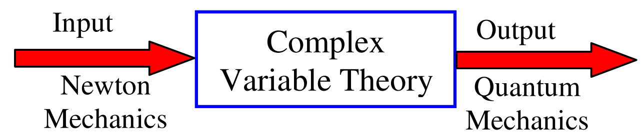

Fig.1 Complex mechanics is an interface between Newton mechanics and quantum mechanics.

The necessity of analyzing quantum phenomena in terms of complex trajectories and complex potentials has long been recognized in many branches of quantum physics. It was known that the solution of the Schrödinger equation could be expressed in terms of complex classical paths out of which wave behavior can be constructed [1]. The studies in chaotic tunneling have also revealed that only by including complex trajectories, can the tunneling effect of transition to classically inaccessible regions be fully explained [2]. Especially, the complex trajectories having no connection with the real manifold, called Laputa branches, play a crucial role to generate chaotic tunneling. As regard to tunneling time, a traversal time can be unambiguously defined as the time spent by a particle between given initial and final positions, moving according to complex trajectories with complex-valued position and momentum [3]. Besides tunneling problems, complex trajectory also has successful applications in scattering problems. A highly accurate approximation of quantum scattering by a hard sphere, valid for complete range of scattering angles, has been proposed using complex-valued angular momentum [4].

A primary motivation of extending standard quantum mechanics to complex domain is the studies of Hamiltonians with complex-valued potentials, which appeared firstly in nuclear physics, and is called optical or average nuclear potentials [5]. Quantum analysis using complex Hamiltonian is not only a mathematical tool but also has concrete physical realization. For example, a delocalization phenomenon was found for a non-Hermite Hamiltonian containing a constant imaginary vector potential [6]. As the imaginary vector potential increases, all of originally localized eigenfunctions get delocalized one by one. This delocalization phenomenon caused by complex potential has a physical realization as flux-line depining in type-II superconductors [7].

Hamiltonian with complex potential has complex eigenfunctions and is non-Hermitian in general, which, at first glance, does not satisfy the Hermitian property, required by the standard quantum mechanics to ensure the reality of energy spectrum. However, the researches of complex potentials have proved the fact that Hermiticity of the Hamiltonian is not essential for a real spectrum. Replacing the Hermiticity condition by a weaker condition called PT symmetry [8], one can obtain new classes of complex Hamiltonians whose spectra are still real and positive.

The relations of Schrödinger equation to particle's motion in complex space have been pointed out by several authors. The first relation comes from a recent discovery about a new interpretation of Schrödinger and Klein-Gordon equations [9], wherein conservation of probability is replaced with conservation of energy in complex domain. The other relation of Schrödinger equation to particle's complex motion stems from the Nelson's derivation of the Schrödinger equation from Newtonian Mechanics [10]. Nelson showed that an entirely Newtonian derivation of the Schrödinger equation could be given by considering Brownian motion with diffusion coefficient $\mathcal{D} = \hbar / 2m$. The most remarkable existing work on complex motion is Nottale's theory of scale relativity [11]. Scale relativity leads naturally to the concept of fractal space-time, which describes quantum space-time as a non-differentiable fractal continuum. Nottale's fractal hypothesis gives rise to the enlightening result that the Nelson's Brownian approach to quantum mechanics can be simply reproduced by replacing the classical time-derivative in Newtonian mechanics by a new complex derivative. The resulting complex Newtonian equation is shown to be exactly identical to the Schrödinger 's equation.

The above-related researches in classical and quantum mechanics in complex domain provide us with good reason to consider seriously the possibility that actual particle motion happens in complex space, but what we have sensed and measured is only the real part of the motion, which constitutes the real physical world that we experience in daily life. Accordingly, the establishment of complex mechanics proposed here is based on such a postulate that the actual scenario of dynamic motion happens in complex space and what we customarily consider as physical reality is merely the projection of the actual scenario into the real space. Within complex domain, we will find that classical mechanics and quantum mechanics can be made compatible with each other and can be further incorporated into a unified framework - called complex mechanics here.

• Complex Mechanics

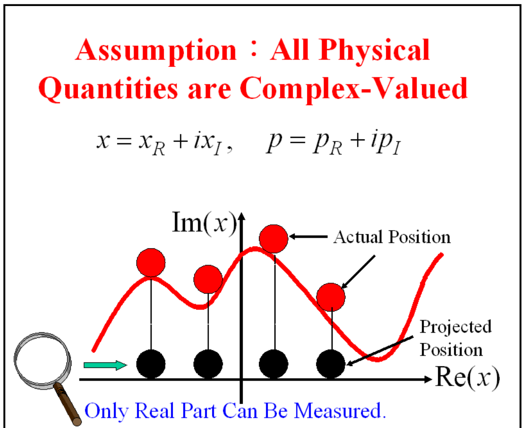

Fig.2 The assumption of complex mechanics.

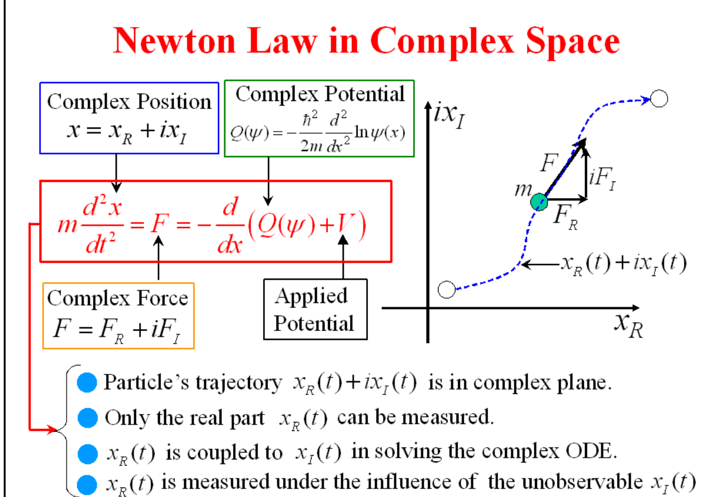

Fig.3 Quantum mechanics can be reformulated by the Newton second law defined in a complex plane wherein a particle is subjected to a complex force and is described by complex coordinate.

Classical mechanics and quantum mechanics were developed according to totally different philosophy, but rigorous proof shows that they are equivalent theories when viewed from complex domain. Complex mechanics is the consequence of such equivalence. Complex mechanics [12] is a new formulation of quantum mechanics, whose main idea is to extend all the physical quantities, such as position, momentum, angular momentum, force, and energy, etc., to a complex domain so as to develop complex-extended Newton mechanics, complex-extended Hamilton mechanics, and complex-extended relativistic mechanics. When we are dealing with the complex-extended Newton mechanics, we naturally arrive at the Schrödinger equation in quantum mechanics; when we are dealing with the complex-extended relativistic equations we naturally obtain the Dirac equation in relativistic quantum mechanics. Under the structure of complex space, the uncertainty in quantum mechanics disappears automatically. The resulting determinism then allows us to integrate quantum mechanics, classical mechanics, and relativistic mechanics in the same framework. Fig.1 illustrates the main feature of complex mechanics as an interface with input from Newton mechanics (classical mechanics) and output to quantum mechanics via the transformation machine – complex variable theory.

Because real space is a subset of complex space, physical laws developed within real space remain valid in complex space. Extending physical laws into a complex domain can widen their validity and explain what cannot be explained in the real space. As Feynman said, quantum mechanics tells us how to compute the motion of a particle but does not tells us the whys to do so. Complex mechanics is just developed to answer the whys that cannot be answered by quantum mechanics. Considering the following problems:

(a) Why does a material particle exhibit wave motion?
(b) Why dose a quantum particle have multiple paths?
(c) Why is probability interpretation unavoidable?
(d) Why must physical quantities in quantum mechanics be defined in terms of their

accompanying operators?

I have proved that all of them are originated from the particle's motion in complex space. Since standard quantum mechanics is defined in real space, it is not surprising that quantum mechanics could not explain the above problems that are caused by particle's motion in complex space.

Complex mechanics is dedicated to the development of a general theory unifying classical mechanics, quantum mechanics, and relativistic mechanics in complex space, and to the conveyance of the philosophy that what have been considered as probabilistic quantum events have a common origin from the particle's deterministic motion in complex space. We postulate that the actual scenario of dynamic motion happens in complex space [13] and what we customarily consider as physical reality is merely the projection of the actual scenario into the real space (refer to Fig.2). The proposed theory employs complex-extended classical mechanics to describe and model quantum systems in such a way that all the particle-like properties can be reserved due to its classical nature and in the meanwhile, all the wave-like properties are manifested naturally via the multi-path behavior of complex trajectories [14].

• Complex Newton Law

The proposed framework of complex mechanics makes use of classical concepts and tools to deal with particle's quantum behavior by the introduction of a complex Hamiltonian from which complex Hamilton equations describing particle's quantum motion are derived in a form of Newton's second law defined in complex space as shown in Fig.3. Distinct from classical Newton second law, the complex Newton law relates complex acceleration to complex force and thus describes a particle's motion in the complex plane.

The complex force appeared in Fig.3 contains the classical force $-dV/dx$ and the quantum force $-dQ/dx$, where Q is the complex potential determined from the wavefunction $\psi(x)$ which is a solution of the Schrödinger equation. Once ψ is given, we then can find the particle's motion by solving the complex Newton equation as shown in Fig.3.

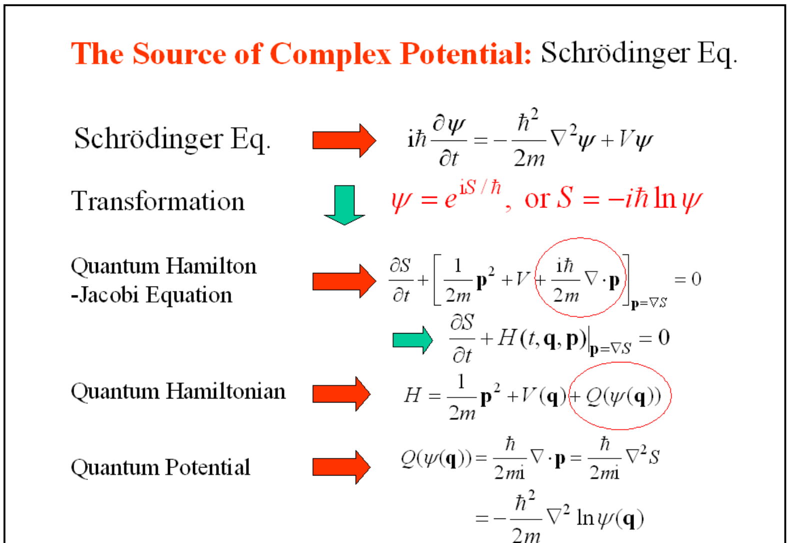

Fig.4 The equivalence between the Schrödinger equation and the quantum Hamilton-Jacobi equation, showing that the complex potential Q is determined from a solution ψ of the Schrödinger equation.

Fig.4 illustrates the procedures for finding the complex potential $Q(\psi(x))$ from the Schrödinger equation. The underlying principle is the equivalence between the Schrödinger equation and the quantum Hamilton-Jacobi equation. From the derived quantum Hamiltonian H, we find that the total energy of a quantum system contains the complex potential Q in addition to the kinetic energy $p^2/2m$ and the applied potential V. It is the action of the complex potential Q that produces the observed quantum phenomena. Substituting the computed Q into the complex Newton equation described in Fig.3, we can find the particle's motion $x(t)$ by integration.

• Comparisons

Fig.5 gives a comparison of the fundamental principles between quantum mechanics and complex mechanics. Quantum mechanics had been established from the following two postulates:

(1) Postulate of correspondence: to any self-consistent and well-defined observables A, there corresponds an operator $\widehat{\mathrm{A}}$.

(2) Postulate of quantization: The operator $\widehat{\mathrm{A}}$ corresponds to the observable $\mathrm{A}(\boldsymbol{q},\boldsymbol{p})$ can be constructed by replacing the coordinate $\boldsymbol{q}$ and momentum $\boldsymbol{p}$ in the expression for A by the

assigned operators $\boldsymbol{q} \to \hat{\boldsymbol{q}} = \boldsymbol{q}$ and $\boldsymbol{p} \to \hat{\boldsymbol{p}} = -i\hbar\nabla$.

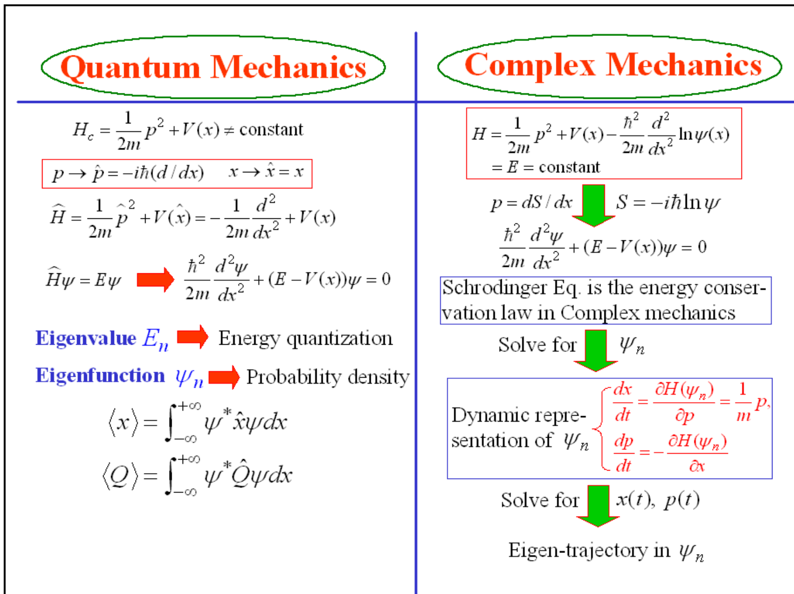

Fig.5 The comparison between quantum mechanics and complex

The validity of the two postulates was justified indirectly via the voluminous precise predictions of quantum mechanics. Although the two postulates work very successfully, until now we still do not know why they should work and a formal proof of their origin still lacks. We also do not know very clearly about the underlying reason that to obtain the correct operators in coordinate system other than Cartesian coordinates, it is always necessary to transform $A(\boldsymbol{q}, \boldsymbol{p})$ into Cartesian coordinates before putting in the operators. I have proved the above two postulates by using complex mechanics [12,15] and also expounded the reason why the postulate of quantization is only true in the Cartesian coordinates. It can be seen from Fig.5 that in complex mechanics we can find the position $x(t)$ and momentum $p(t)$ in the quantum state ψ_n as a function of time t, while in quantum mechanics only the mean values $\langle x \rangle$ and $\langle p \rangle$ can be obtained. In other words, given a wavefunction ψ, we can determine not only the probability density $\psi^*\psi$ but also a dynamic representation for ψ as shown in Fig.5, from which dynamic responses $x(t)$ and $p(t)$ can be found.

I have applied complex mechanics to solve many quantum problems by using common methods in engineering mechanics with very promising results. The following summarizes some of them:

(1) Relate complex variables to their accompanying quantum operators and show every quantum operator appeared in quantum mechanics can be explicitly represented by its associated complex variable [12,15].
(2) Explain and solve quantum tunneling problem by complex Newton equation [16].
(3) Formulate and solve quantum motions in diatomic molecules by complex Newton equation [17,18].
(4) Demonstrate how a material particle can produce wave motion and why it traces multiple paths [14,19].
(5) Prove that the shell structure found in the hydrogen atom is due to the action of the complex quantum potential Q [20].
(6) Prove the noticeable fact that the electron spin is actually induced by its motion in complex space [21,22].
(7) Compute the quantum scattering trajectories of an electron incident on a proton by complex Newton equation [23].
(8) Analyze quantum transition behavior by complex Newton second law [24].

The above successful demonstrations of solving quantum problems by classical methods will form a solid foundation for the course of engineering quantum mechanics to be introduced below.

• Teaching Plan

Complex mechanics shortens the gap between quantum mechanics and engineering mechanics and makes the teaching and learning of quantum mechanics easier. The incorporation of complex mechanics into the current engineering education in NCKU would be very helpful to our students, making them more powerful in handling problems relating to nanotechnology and biotechnology. The proposed teaching program has been tested in the course named “Engineering Quantum Mechanics” in the Institute of Aeronautics and Astronautics at the semester year from 2006 to 2007.

The preliminary validation of the idea of teaching quantum mechanics fully with engineering mechanics is very successful in this course. Students enrolled in this course have been benefited greatly from the bridge provided by complex mechanics that allows them to accelerate and deepen the learning of quantum mechanics by their previous knowledge and experience gained from the course of engineering mechanics. Teaching experience learned from this preliminary course further improves the teaching plan for the next semester. Following is the proposed teaching plan for the coming semester.

Lecture No.	Schedule (3hr/week)	Topics
1	1st week	Review of Newton, Lagrange, and Hamilton formulations of classical mechanics.
2	2nd week	Review of complex variable theory.
3	3rd week	Overview of quantum mechanics: quantum operator and eigenvalue problems
4	4th week	Fundamentals of complex mechanics
5	5th week	Relate a complex variable to its accompanying quantum operator
6	6th week	Solve quantum harmonic oscillator by complex Newton second law
7	7th week	Produce wave-particle duality by particle's motion in complex plane
8	8th week	Describe quantum vibrational dynamics in diatomic molecules by complex Newton laws
9	9th ~10th week	Describe tunneling dynamics by complex mechanics
10	11th week	Generate Feynman's multiple paths by complex Newton equations.
11	12th week	Define orbital and spin angular momentum in a engineering sense
12	13th ~14th week	Solve electronic quantum motions in hydrogen atom by Hamilton equations of motion
13	15th week	Compute quantum scattering trajectory of an electron incident on a proton by Hamilton equations of motion.
14	16th week	Quantum dynamics in electromagnetic fields.
15	17th ~18th week	Electronic quantum motions in quantum wires and quantum dots.

• Anticipated Contributions

The success of the preliminary teaching experiment gave me the confidence that every student with engineering background in NCKU can familiarize himself with quantum mechanics fully by their previous learning experience, if lecturers of engineering mechanics would take 3~6 hours in their courses to introduce the interface with quantum mechanics via the introduction of engineering quantum mechanics. NCKU will be the first university in the world to claim the unification of quantum mechanics and engineering mechanics in its engineering education. With the execution of this project, I want to share the delight of teaching engineering quantum mechanics with my colleagues in NCKU and to promote the usefulness of engineering quantum mechanics in the modern engineering education in NCKU and other universities in the world. The outcome of this project will be a deliberately prepared and practically tested lecture note (with length about 500 typing pages), which, I think, has the potential to become a leading textbook in the field of modern engineering education.

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February 2008