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Invariant Eigen-Structure in Complex-Valued Quantum Mechanics

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Abstract

The complex-valued quantum mechanics considers quantum motions on a complex plane instead of on a real axis, and presents variations of a particle's complex position, momentum and energy along a complex trajectory. On the basis of quantum Hamilton-Jacobi formalism in the complex space, we point out that having complex-valued motions is a universal property for quantum systems because every quantum system is actually accompanied with an intrinsic complex Hamiltonian derived from the Schrödinger equation. It is revealed that the conventional real-valued quantum mechanics is a special case of the complex-valued quantum mechanics in which eigen-structures of real and complex quantum systems, such as their eigenvalues, eigenfunctions and eigen-trajectories, are invariant under linear complex mapping. In other words, there is indeed no distinction between Hermitian, PT-symmetric, and non PT-symmetric systems when viewed from a complex domain. Their eigen-structures can be made coincident through linear transformations of complex coordinates.

Keywords: Complex Hamiltonian; Complex Bohmian mechanics; Intrinsic Hamiltonian

1. Introduction

Hermiticity as a critical property providing the reality of spectrums had been replaced by a weaker property called PT-symmetry [1]. Recently, complex spectrums in PT-symmetric quantum mechanical systems have brought a resurgence of interest in discussing the complex extension [2]. It brings effects on the discussion of complex properties of quantum mechanical systems, and gains attention to the exploration of the reality reflected by complex features of the microscopic world [3,4]. Some classical approaches to those complex issues have been proposed [5,6], in which they embed classical trajectories in a complex space .

The necessity of extending quantum mechanics to a complex domain comes from the observation that the Schrödinger equation can be transformed into a quantum Hamilton-Jacobi (H-J) equation, wherein canonical variables are complex-valued [7]. Based on the quantum H-J equation, quantum Hamilton mechanics was established in [8,9], where a fully classical interpretation of quantum mechanics was proposed in a complex domain. It was pointed out that the wave-particle duality, the uncertainty principle, and the Feynman's multi-path phenomenon all originate from the projection effect from the complex space to the real space [10-12]. Strong evidences from the El Naschie's *E*-infinity theory [13,14] and the many other accompanied works [15-17] further indicate that quantum phenomena are produced by the non-classical topology and geometry of quantum spacetime when projected into our 3+1 Euclidean space [18-20].

By using the complex-space formulation of fractal spacetime [21-22], many quantum phenomena, such as tunneling [23], wave-particle duality [10], spin [24,25], state transition [26], path integral [12], quantum chaos [27], uncertainty principle [11] and molecular dynamics [28,29], have been modeled exactly in the framework of complex-extended Hamilton mechanics, wherein quantum motions are