Stability and quantization of complex-valued nonlinear quantum systems

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Abstract

In this paper, we show that quantum mechanical systems can be fully treated as complex-extended nonlinear systems such that stability and quantization of the former can be completely analyzed by the existing tools developed for the latter. The concepts of equilibrium points, index theory and Lyapunov stability theory are all extended to a complex domain and then used to determine the stability of quantum mechanical systems. Modeling quantum mechanical systems by complex-valued nonlinear equations leads naturally to the phenomenon of quantization. Based on the residue theorem, we show that the quantization of a physical quantity $f(x,p)$ is a consequence of the trajectory independence of its time-averaged mean value $\langle f(x,p)\rangle_{x(t)}$. Three types of trajectory independence are observed in quantum systems. Local and global trajectory independences correspond to the quantizations of $\langle f(x,p)\rangle_{x(t)}$ within a given state ψ, while universal trajectory independence implies that $\langle f(x,p)\rangle_{x(t)}$ is further independent of the quantum state ψ. By using the property of universal trajectory independence, we give a formal proof of the Bohr–Sommerfeld

quantization postulate $\int p dx = nh$ and the Planck–Einstein quantization postulate $E = nh\nu$, $n = 0,1,\ldots$.