A new hydrodynamic formulation of complex-valued quantum mechanics

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Abstract

In this paper, a new hydrodynamic formulation of complex-valued quantum mechanics is derived to reveal a novel analogy between the probability flow and the potential flow on the complex plane. For a given complex-valued wavefunction $\Psi(z,t)$, $z = x + \mathrm{i}y \in \mathbb{C}$, we first define a complex potential function $\Omega(z,t) = \hbar/(\mathrm{i}m) \ln\Psi(z,t) = \phi(x,y,t) + \mathrm{i}\psi(x,y,t)$ with $x, y \in \mathbb{R}$ and then prove that the streamline lines $\psi(x,y,t) = c_\psi$ and the potential lines $\phi(x,y,y) = c_\phi$ in the potential flow defined by Ω are equivalent to the constant-probability lines $|\Psi| = c_1$ and the constant-phase lines $\angle\Psi = c_2$ in the probability flow defined by Ψ. The discovered analogy is very useful in visualizing the unobservable probability flow on the complex $x + \mathrm{i}y$ plane by analogy with the 2D potential flow on the real $x - y$ plane, which can be visualized by using dye streaks in a fluid laboratory.