Strong chaos in one-dimensional quantum system

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Abstract

According to the Poincaré–Bendixson theorem, a minimum of three autonomous equations is required to exhibit deterministic chaos. Because a one-dimensional quantum system is described by only two autonomous equations using de Broglie–Bohm's trajectory interpretation, chaos in one-dimensional quantum systems has long been considered impossible. We will prove in this paper that chaos phenomenon does exist in one-dimensional quantum systems, if the domain of quantum motions is extended to complex space by noting that the quantum world is actually characterized by a four-dimensional complex spacetime according to the $E^{(\infty)}$ theory. Furthermore, we point out that the interaction between the real and imaginary parts of complex trajectories produces a new chaos phenomenon unique to quantum systems, called strong chaos, which describes the situation that quantum trajectories may emerge and diverge spontaneously without any perturbation in the initial position.