The origin and proof of quantization axiom $\mathbf{p} \to \hat{\mathbf{p}} = -i\hbar\nabla$ in complex spacetime

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Abstract

The quantization axiom $\dot{p} = -\frac{\partial H}{\partial q} = -\frac{\partial}{\partial q}(V + Q)$ is the kernel in constructing quantum-mechanical systems; however, it was proposed without proof and even till now no formal proof has been given about its origin and validity by using fundamental theory of mechanics. This paper aims to show that quantum operators have the root in complex spacetime and can be derived naturally from complex-extended Hamilton equations of motion. The derivation of quantum operators from Hamilton mechanics is coordinate-independent and thus allows us to deduce operators directly from any curved spacetime without transforming back to Cartesian space.