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Nonlinear Dynamics Governing Quantum Transition Behavior

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Abstract

In quantum mechanics, the only known quantitative index describing state transition process is the occupation probability. In this paper, we propose three new quantitative indices, namely, complex transition trajectory, quantum potential, and total quantum energy, to give an elaborate description of state-transition behavior, which otherwise cannot be manifested in terms of occupation probability. The new quantitative indices and the nonlinear dynamics governing quantum transition behavior are derived from the quantum Hamilton mechanics that provides a unified description of the entire transition process from an initial state to a final state via an entangled state. Under the framework of quantum Hamilton mechanics, state transition behavior, which was originally represented by the probabilistic view, can now be described deterministically in terms of the continuity of total energy, of quantum potential, and of complex trajectory. The proposed nonlinear analysis of transition behavior is then applied to harmonic oscillator, entangled free-particle motion, and hydrogen atom to demonstrate the usefulness of the new quantitative indices in characterizing the progression of state transition process.

Keywords: quantum transition, quantum Hamilton mechanics, entangled state, nonlinear quantum dynamics

1. Introduction

The last three decades have witnessed growing importance on the research of quantum entanglement, which is the core of many applications, such as quantum computation, quantum teleportation, quantum communication, dense coding and chemical reaction control, etc [1-3]. Entanglement is a characteristic of quantum mechanics, describing correlations between quantum systems and having no exact classical analog [4]. Transition between two discrete states has ever been described by two-state models under certain conditions [5]. Several studies [6,7] have suggested the feasibility of classical entanglement model by examining the classical analog of entanglement through Bohmian mechanics [8,9].

On deriving the nonlinear dynamics governing state transition behavior, one is confronted with the problem of how to describe quantum system deterministically. In this aspect, a number of issues have appeared on attempting to connect quantum mechanics to classical mechanics. Under the framework of E-infinity Cantorian spacetime [10-12], quantum motion may be modeled as deterministic chaos. On the other hand, the concept of complex canonical coordinates had been proposed in an early work to search for the connection between classical and quantum mechanics [13]. Relation between Hamilton-Jacobi (H-J) formalism and quantum mechanics was discussed later [14,15]. Meanwhile, some researchers focused on the trajectories in complex domain [16,17]. Recently, a substantial body of researches documents the necessity of representing quantum world in complex domain along the line of non-Hermitian Hamiltonian approach, wherein the conditions of PT symmetry, pseudo-Hermiticity and CPT symmetry have been proposed in sequence to guarantee real spectral energy for complex Hamiltonian [18,19].

A thorough study of the role of complex Hamiltonian in quantum mechanics further led to the groundwork of quantum Hamilton mechanics. It has been shown [20] that quantum mechanics becomes a part of analytical mechanics and can be treated integrally with classical mechanics, if one extends Hamilton