

# Solving Quantum Trajectories in Coulomb Potential by Quantum Hamilton–Jacobi Theory

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*Received 14 September 2005; accepted 3 October 2005*

*Published online 5 December 2005 in Wiley InterScience (www.interscience.wiley.com).*

*DOI 10.1002/qua.20878*

**ABSTRACT:** We show that the quantum central-force problems can be modeled and solved exactly by quantum Hamilton–Jacobi formulation, from which the quantum operators  $\hat{L}_z$ ,  $\hat{L}^2$ , and  $\hat{H}$  can be derived without using the quantization principle  $p \rightarrow (\hbar/i)\partial/\partial x$ . Quantum conservation laws expressed by the Poisson bracket show that the eigenvalues of these quantum operators are just equal to the constants of motion along the eigen-trajectories defined in a complex domain. The shell structure observed in bound systems, such as the hydrogen atom, is found to stem from the structure of the quantum potential, by which the quantum forces acting on the electron can be uniquely determined, the stability of atomic configuration can be justified, and the quantum trajectories of the electron can be obtained by integrating the related quantum Lagrange equations. On solving the quantum equations of motion, the solution of the Schrödinger equation serves as the first integration of the second-order quantum Lagrange equations. The stable equilibrium points of the derived first-order nonlinear quantum dynamics are shown to be identical to the positions with maximum probability predicted by standard quantum mechanics. The internal mechanism of how the quantum dynamics evolve continuously to classical dynamics and of how the quantum conservation laws transit continuously to the classical conservation laws as  $n \rightarrow \infty$  are analyzed in detail. The construction of the quantum scattering trajectory by searching for an unbound solution for the Schrödinger equation is investigated. © 2005 Wiley Periodicals, Inc. *Int J Quantum Chem* 106: 1620–1639, 2006

**Key words:** quantum motion; Coulomb potential; quantum Hamilton–Jacobi theory; complex domain