Quantum Hamilton mechanics: Hamilton equations of quantum motion, origin of quantum operators, and proof of quantization axiom

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Abstract

This paper gives a thorough investigation on formulating and solving quantum problems by extended analytical mechanics that extends canonical variables to complex domain. With this complex extension, we show that quantum mechanics becomes a part of analytical mechanics and hence can be treated integrally with classical mechanics. Complex canonical variables are governed by Hamilton equations of motion, which can be derived naturally from Schrödinger equation. Using complex canonical variables, a formal proof of the quantization axiom $\mathbf{p} \rightarrow \hat{\mathbf{p}} = -i\hbar\nabla$, which is the kernel in constructing quantum-mechanical systems, becomes a one-line corollary of Hamilton mechanics. The derivation of quantum operators from Hamilton mechanics is coordinate independent and thus allows us to derive quantum operators directly under any coordinate system without transforming back to Cartesian coordinates. Besides deriving quantum operators, we also show that the various prominent quantum effects, such as quantization, tunneling, atomic shell structure, Aharonov–Bohm effect, and spin, all have the root in Hamilton mechanics and can be described entirely by Hamilton equations of motion.

Keywords: Hamilton mechanics; Complex extension; Quantum motion; Quantum operator; Quantization axiom