Quantum dynamics of hydrogen atom in complex space

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Abstract

We show in this paper that the electron's quantum dynamics in hydrogen atom can be modeled exactly by quantum Hamilton–Jacobi formalism. It is found that the quantizations of energy, angular momentum, and the action variable $\int p\,dq$ are all originated from the electron's complex motion, and that the shell structure observed in hydrogen atom is indeed originated from the structure of the complex quantum potential, from which the quantum forces acting upon the electron can be uniquely determined, the stability of atomic configuration can be justified, and the electron's complex trajectories can be derived accordingly. Based on the derived electron's trajectory, we can explain why the electron appears at some positions with large probability, while at some other positions with small probability. The positions with maximum probability predicted by standard quantum mechanics are found to be just the stable equilibrium points of the electron's non-linear complex dynamics. The electron's trajectories in hydrogen atom are discovered to be very diverse and strongly state-dependent; some of them are open and non-periodic, while some are closed and periodic. Over such a great diversity of orbits, commensurability condition ensuring the existence of closed orbit will be derived and the de Broglie's standing wave pattern will be identified. Along the investigation of the electron's orbits in hydrogen atom, we will also clarify why old quantum mechanics using the concept of classical orbit can correctly predict the energy quantization of hydrogen atom and meanwhile why it is not applicable to general quantum system. Finally, the internal mechanism of how the precessing, non-conical eigen-trajectories can evolve continuously to the

classical, non-precessing, conical orbits as $n \to \infty$ is explained in detail.

Article Outline